§5.3 **Name:** ____________________

Fall MATH 1120 Lec003 29 August - 2 September 2022

# Integration by Substitution

## Review

- **Chain rule:** Suppose $u$ is differentiable at $x$ and $f$ is differentiable at $u(x)$. Then the derivative of the composite function $f(u(x))$ is given by

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[f(u(x))\right] = f'(u(x))u'(x)$$

- $\frac{\mathrm{d}}{\mathrm{d}x}\tan x = \sec^2 x \quad \frac{\mathrm{d}}{\mathrm{d}x}\cot x = -\csc^2 x \quad \frac{\mathrm{d}}{\mathrm{d}x}\sec x = \sec x \tan x \quad \frac{\mathrm{d}}{\mathrm{d}x}\csc x = -\csc x \cot x$
- $\frac{\mathrm{d}}{\mathrm{d}x}\arcsin x = \frac{1}{\sqrt{1-x^2}} \quad \frac{\mathrm{d}}{\mathrm{d}x}\arctan x = \frac{1}{1+x^2}$

## Main Concepts

- **Integrating a function composed with a linear function:** Let $h(x) = f(ax+b)$ and let $F$ denote the (algebraic) antiderivative of $f$, then the (general) antiderivative of $h$ is given by:

$$H(x) = \frac{1}{a}F(ax+b) + C.$$

- **Integration by Substitution:** Suppose $g$ is a differentiable function with continuous derivative, $g'$ and $f$ is a continuous function. Then,

$$\int f'(g(x))g'(x)\,\mathrm{d}x = \int f'(u)\,\mathrm{d}u = f(u) + C = f(g(x)) + C,$$

with the substitution $u = g(x)$.

## Activities

### Activity 5.3.2

Evaluate each of the following indefinite integrals. Check each antiderivative that you find by differentiating.

(a) $\int \sin(8 - 3x)\, \mathrm{d}x$

(b) $\int \sec^2(4x)\, \mathrm{d}x$

(c) $\int \frac{1}{11x-9}\, dx$

(d) $\int \csc(2x + 1) \cot(2x + 1)\, \mathrm{d}x$

(e) $\int \frac{1}{\sqrt{1-16x^2}}\, \mathrm{d}x$

(f) $\int 5^{-x}\, \mathrm{d}x$

Activity 5.3.3

Evaluate each of the following indefinite integrals by using these steps:

- Find two functions within the integrand that form (up to a possible missing constant) a function-derivative pair;
- Make a substitution and convert the integral to one involving $u$ and $du$;
- Evaluate the new integral in $u$;
- Convert the resulting function of $u$ back to a function of $x$ by using your earlier substitution;
- Check your work by differentiating the function of $x$. You should come up with the integrand originally given.

(a) $\int \frac{x^2}{5x^3+1}\,\mathrm{d}x$

(b) $\int e^x \sin(e^x)\,\mathrm{d}x$

(c) $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,\mathrm{d}x$

Activity 5.3.4

Evaluate each of the following definite integrals exactly through an appropriate $u$-substitution.

(a) $\int_1^2 \frac{x}{1+4x^2}\,\mathrm{d}x$

(b) $\int_0^1 e^{-x}(2e^{-x}+3)^9\,\mathrm{d}x$

(c) $\int_{2/\pi}^{4/\pi} \frac{\cos\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x$