§5.4 **Name:** ______________________

Fall MATH 1120 Lec003 29 August - 2 September 2022

Integration by Parts

Review

- **Product Rule:** Let u and v be differentiable functions. Then the derivative of the product is given by

$$\frac{\mathrm{d}}{\mathrm{d}x}(u \cdot v) = u \cdot \frac{\mathrm{d}v}{\mathrm{d}x} + \frac{\mathrm{d}u}{\mathrm{d}x} \cdot v$$

Main Concepts

- **Integrating by parts** can be used to compute integrals of the form $\int f(x)g'(x)\,\mathrm{d}x$:

$$\int f(x)\,g'(x)\,\mathrm{d}x = f(x)\,g(x) - \int g(x)\,f'(x)\,\mathrm{d}x$$

It is also expressed as

$$\int u\,\mathrm{d}v = u\,v - \int v\,\mathrm{d}u$$

where $u = f(x)$, $\mathrm{d}u = f'(x)\,\mathrm{d}x$, $v = g(x)$ and $\mathrm{d}v = g'(x)\,\mathrm{d}x$.

Activities

Activity 5.4.2

Evaluate each of the following indefinite integrals. Check each antiderivative that you find by differentiating.

(a) $\int te^{-t}\,\mathrm{d}t$

(b) $\int 4x\sin(3x)\,\mathrm{d}x$

(c) $\int z\sec^2(z)\,\mathrm{d}z$

(d) $\int x\ln(x)\,\mathrm{d}x$

Activity 5.4.2

Evaluate each of the following indefinite integrals, using the provided hints.

(a) Evaluate $\int \arctan(x)\,\mathrm{d}x$ by using Integration by Parts with the substitution $u = \arctan(x)$ and $dv = 1\,\mathrm{d}x$.

(b) Evaluate $\int \ln(z)\,\mathrm{d}z$. Consider a similar substitution to the one in (a).

(c) Use the substitution $z = t^2$ to transform the integral $\int t^3 \sin(t^2)\,\mathrm{d}t$ to a new integral in the variable z, and evaluate the new integral by parts.

(d) Evaluate $\int s^5 e^{s^3} \, \mathrm{d}s$ using an approach similar to that described in (c).

(e) Evaluate $\int e^{2t} \cos(e^t) \, \mathrm{d}t$. You will find it helpful to note that $e^{2t} = e^t \cdot e^t$.

ACTIVITY 5.4.2

Evaluate each of the following indefinite integrals.

(a) $\int x^2 \sin(x) \, \mathrm{d}x$

(b) $\int t^3 \ln(t) \, \mathrm{d}t$

(c) $\int e^z \sin(z)\,\mathrm{d}z$

(d) $\int s^2 e^{3s}\,\mathrm{d}s$

(e) $\int t \arctan(t)\,\mathrm{d}t$ (**Hint:** At a certain point in this problem, it is very helpful to note that $\frac{t^2}{1+t^2} = 1 - \frac{1}{1+t^2}$.)