§5.6 **Name:** ____________________

Fall MATH 1120 Lec003 5 September - 9 September 2022

# Numerical Integration

## Main Concepts

- Numerical algorithms are useful for approximating the values of definite integrals.
- Left-handed Riemann sum:

$$L_n = \Delta x(f(x_0) + f(x_1) + ... + f(x_{n-1}))$$

- Right-handed Riemann sum:

$$R_n = \Delta x(f(x_1) + f(x_2) + ... + f(x_n))$$

- Midpoint Rule:

$$M_n = \sum_{i=1}^{n} f\left(\frac{x_{i-1} + x_i}{2}\right)\Delta x$$

- Trapezoid rule:

$$T_n = \sum_{i=0}^{n-1} \frac{f(x_i) + f(x_{i+1})}{2}\Delta x$$

- Simpson's rule:

$$S_{2n} = \frac{2M_n + T_n}{3}$$

## Activities

### Activity 5.6.2

In this activity, we explore the relationships among the errors generated by left, right, midpoint, and trapezoid approximations to the definite integral $\int_1^2 \frac{1}{x^2}\,dx$.

(a) Use the first FTC to evaluate $\int_1^2 \frac{1}{x^2}\,dx$ exactly.

$\int_1^2 \frac{1}{x^2}dx = -\frac{1}{x}\Big|_1^2 = -\frac{1}{2} - \left(\frac{-1}{1}\right)$

$= \frac{-1}{2} + 1 = \frac{1}{2}$

$= 0.5$

(b) Use appropriate computing technology to compute the following approximations for $\int_1^2 \frac{1}{x^2}\,dx : T_4, M_4, T_8$, and $M_8$.

$T_4 = 0.50899$ $T_8 = 0.50227$

$M_4 = 0.49555$ $M_8 = 0.49887$

(c) Let the *error* that results from an approximation be the approximation's value minus the exact value of the definite integral. For instance, if we let $E_{T,4}$ represent the error that results from using the trapezoid rule with 4 subintervals to estimate the integral, we have

$$E_{T,4} = T_4 - \int_1^2 \frac{1}{x^2}\,dx.$$

Similarly, we compute the error of the midpoint rule approximation with 8 subintervals by the formula

$$E_{M,8} = M_8 - \int_1^2 \frac{1}{x^2}\,dx.$$

Based on your work in (a) and (b) above, compute $E_{T,4}, E_{T,8}, E_{M,4}, E_{M,8}$.

$E_{T,4} = 0.00899$ $E_{M,4} = 0.00445$

$E_{T,8} = 0.00227$ $E_{M,8} = 0.00113$

(d) Which rule consistently over-estimates the exact value of the definite integral? Which rule consistently under-estimates the definite integral?

T overestimates

M underestimates

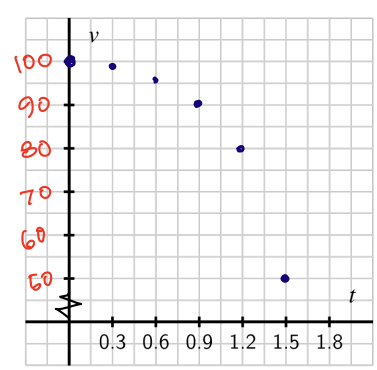
(e) What behavior(s) of the function $f(x) = \frac{1}{x^2}$ lead you to your observations in (d)?

$\frac{1}{x^2}$ is concave up on $[1, 2]$

ACTIVITY 5.6.3

A car traveling along a straight road is braking and its velocity is measured at several different points in time, as given in the following table. Assume that $v$ is continuous, always decreasing, and always decreasing at a decreasing rate, as is suggested by the data.

| seconds, $t$ | Velocity in ft/sec, $v(t)$ |
|---|---|
| 0 | 100 |
| 0.3 | 99 |
| 0.6 | 96 |
| 0.9 | 90 |
| 1.2 | 80 |
| 1.5 | 50 |
| 1.8 | 0 |




(a) Plot the given data on the set of axes provided with time on the horizontal axis and the velocity on the vertical axis.

(b) What definite integral will give you the exact distance the car traveled on $[0, 1.8]$?

$$\int_0^{1.8} v(t)\,dt$$

(c) Estimate the total distance traveled on $[0, 1.8]$ by computing $L_3$, $R_3$ and $T_3$. Which of these under-estimates the true distance traveled?

$L_3 = 165.6$ ft $\quad R_3 = 105.6$ ft

$T_3 = 135.6$ ft

$R_3, T_3 \rightarrow$ underestimate (from concave down)

(d) Estimate the total distance traveled on $[0, 1.8]$ by computing $M_3$. Is this an over- or under-estimage? Why?

$M_3 = 143.4$ ft $\quad$ overestimate (concave down)

(e) Using your results from (c) and (d), improve your estimate further by using Simpson's Rule $S_{2n} = \frac{2M_n + T_n}{3}$.

$S_6 = 140.8$ ft

(f) What is your best estimate of the average velocity of the car on $[0, 1.8]$? Why? What are the units on this quantity?

Simpson's rule is the best approximation.

$\int_0^{1.8} v(t)\,dt \approx 140.8$ ft