§6.2 **Name:** ______________________

Fall MATH 1120 Lec003 12 September - 16 September 2022

Solids of Revolution and their Volumes

Main Concepts

- Solids of Revolution are obtained by revolving curves around a fixed axis.
- **Disk method:** Used to compute volumes of solids generated by revolving a single curve. Suppose $y = r(x)$ is a nonnegative continuous function on $[a, b]$, then the volume of the solid of revolution given by revolving the curve about the x-axis over the interval is given by:

$$V = \int_a^b \pi r(x)^2 \, \mathrm{d}x$$

- **Washer method:** Suppose $R(x)$ and $r(x)$ are nonnegative continuous functions on $[a, b]$ that satisfy $R(x) \geq r(x)$ for all x in $[a, b]$, the solid of revolution has volume

$$V = \int_a^b \pi [R(x)^2 - r(x)^2] \, \mathrm{d}x$$

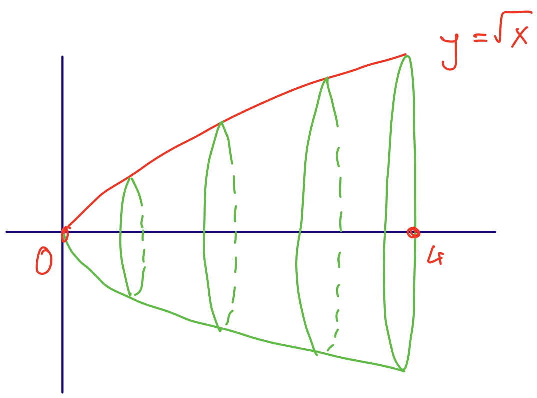
- When the curve is revolved around a different axis, say $y = a$, the volume of the solid of revolution can be computed after shifting the curve by a, viz., $R(x - a)$ and $r(x - a)$.

Activities

Activity 6.2.2

In each of the following questions, draw a careful, labeled sketch of the region described, as well as the resulting solid that results from revolving the region about the stated axis. In addition, draw a representative slice and state the volume of that slice, along with a definite integral whose value is the volume of the entire solid. It is not necessary to evaluate the integrals you find.

(a) The region S bounded by the x-axis, the curve $y = \sqrt{x}$, and the line $x = 4$; revolve S about the x-axis.

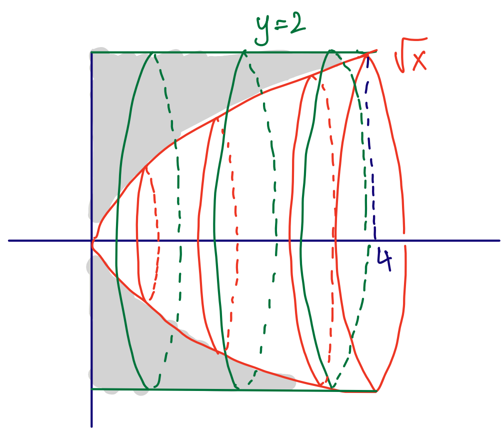


Disk method:

$$V = \pi \int_0^4 \left(\sqrt{x}\right)^2 dx$$

$$= \pi \int_0^4 x \, dx = 8\pi$$

(b) The region S bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 2$; revolve S about the x-axis.

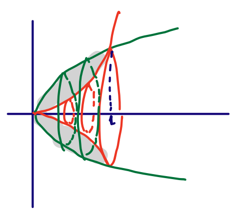


Washer method:

$$\pi \int_0^4 (2)^2 - \left(\sqrt{x}\right)^2 dx = \pi \int_0^4 4 - x \, dx$$

$$= 8\pi$$

(c) The finite region S bounded by the curves $y = \sqrt{x}$ and $y = x^3$; revolve S about the x-axis.

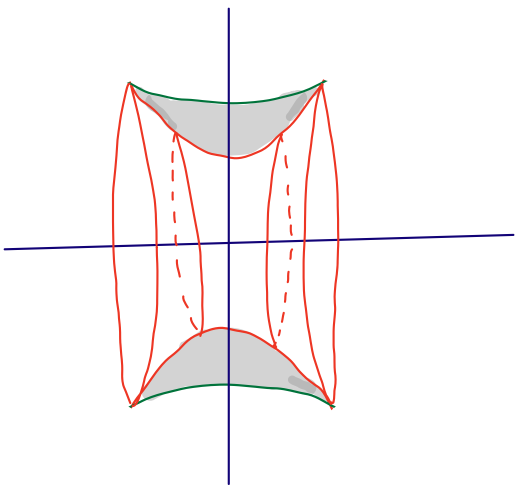


$\sqrt{x} = x^3 \Rightarrow x = x^6 \quad x^6 - x = 0$

$x = 0 \quad x = 1$

$$V = \int_0^1 \pi (x - x^6) \, dx = \frac{5}{14}\pi$$

(d) The finite region S bounded by the curves $y = 2x^2 + 1$ and $y = x^2 + 4$; revolve S about the x-axis.

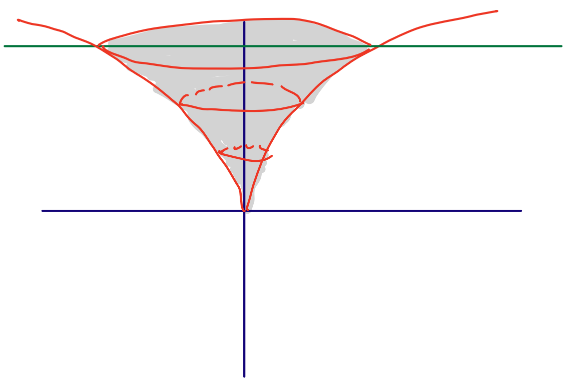


$2x^2+1 = x^2+4$

$x^2 = 3 \qquad x = \pm\sqrt{3}$

$$\pi \int_{-\sqrt{3}}^{\sqrt{3}} (x^2+4)^2 - (2x^2+1)^2 \, dx = \frac{136\sqrt{3}}{5}\pi$$

(e) The region S bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 2$; revolve S about the y-axis. How is this problem different from the one posed in part (b)?

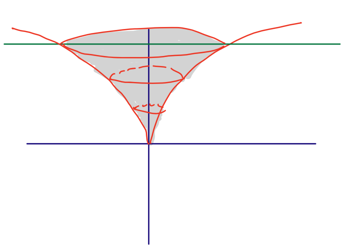


$$\int_0^2 \pi (y^2)^2 \, dy = \frac{32}{5}\pi$$

Activity 6.2.3

In each of the following questions, draw a careful, labeled sketch of the region described, as well as the resulting solid that results from revolving the region about the stated axis. In addition, draw a representative slice and state the volume of that slice, along with a definite integral whose value is the volume of the entire solid. It is not necessary to evaluate the integrals you find.

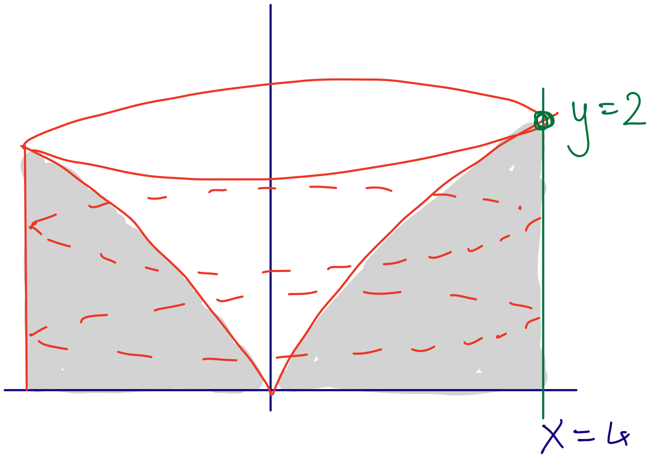
(a) The region S bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 2$; revolve S about the y-axis.



Same as above

(b) The region S bounded by the ~~y-axis,~~ x-axis the curve $y = \sqrt{x}$, and the line $x = 4$; revolve S about the y-axis.

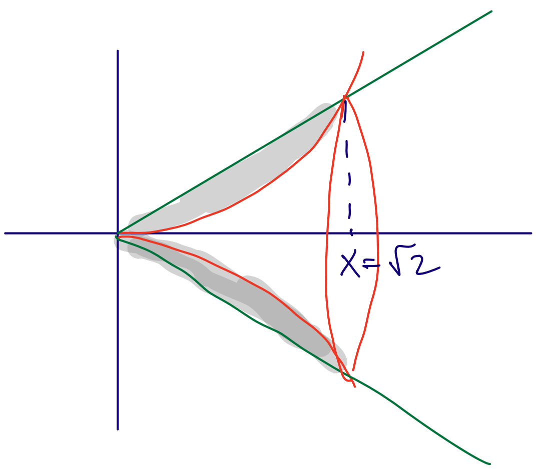
Washer: $R(y) = 4 \quad r(y) = y^2$



$$\pi \int_0 (4)^2 - (y^2)^2 \, dy$$

(c) The finite region S in the first quadrant bounded by the curves $y = 2x$ and $y = x^3$; revolve S about the x-axis.

$2x = x^3 \qquad x = 0$ or $x = \sqrt{2}$ (first quadrat)

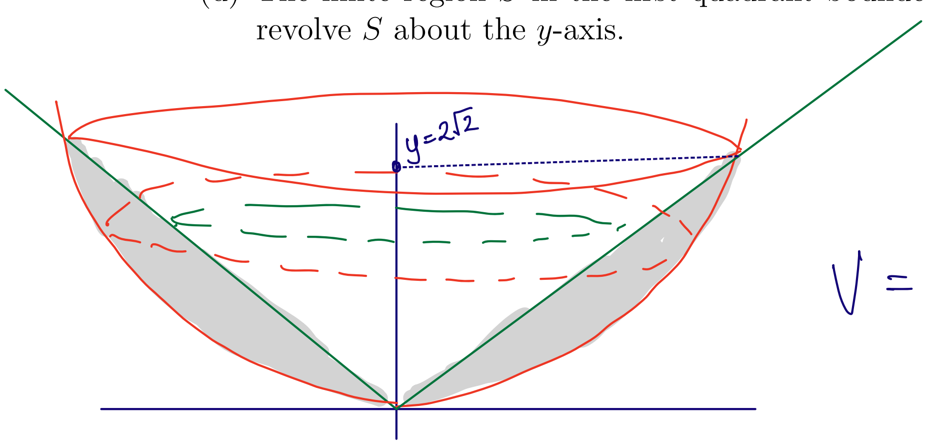


$$\pi \int_0^{\sqrt{2}} (2x)^2 - (x^3)^2 \, dx$$

(d) The finite region S in the first quadrant bounded by the curves $y = 2x$ and $y = x^3$; revolve S about the y-axis.

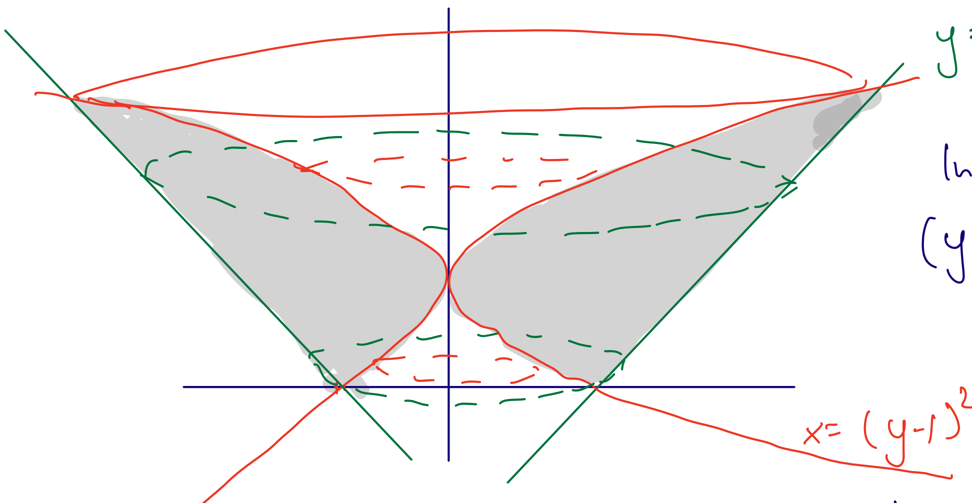
Outer radius: $x = \sqrt[3]{y}$

Inner radius: $x = y/2$



$$V = \pi \int_0^{2\sqrt{2}} \left((\sqrt[3]{y})^2 - (y/2)^2 \, dy \right.$$

(e) The finite region S bounded by the curves $x = (y-1)^2$, and $y = x-1$; revolve S about the y-axis.



Outer : $x = y+1$

Inner : $x = (y-1)^2$

Intersection:

$(y-1)^2 = y+1 \qquad y^2-2y+\cancel{1} = y+\cancel{1}$

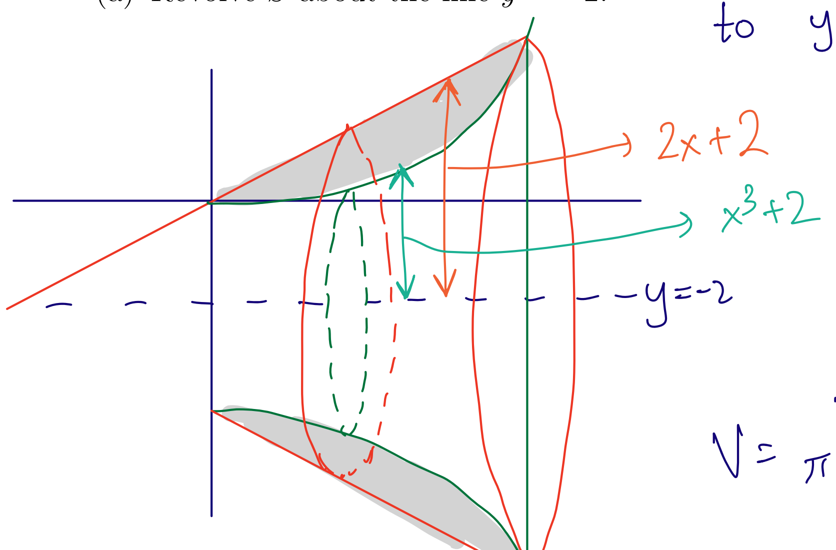
$y^2-3y=0 \qquad y=3$ or $y=0$

$$V = \pi\int_0^3 (y+1)^2 - \left((y-1)^2\right)^2 dy$$

Activity 6.2.4

In each of the following questions, draw a careful, labeled sketch of the region described, as well as the resulting solid that results from revolving the region about the stated axis. In addition, draw a representative slice and state the volume of that slice, along with a definite integral whose value is the volume of the entire solid. It is not necessary to evaluate the integrals you find. For each prompt, use the finite region S in the first quadrant bounded by the curves $y = 2x$ and $y = x^3$.

(a) Revolve S about the line $y = -2$.



We measure radii with respect to $y=-2$

$R(x) = 2x+2$

$r(x) = x^3+2$

Intersection: $2x = x^3$

$x=0$ or $x=\sqrt{2}$

$$V = \pi\int_0^{\sqrt{2}} (2x+2)^2 - (x^3+2)^2 dx$$

(b) Revolve S about the line $y = 4$.

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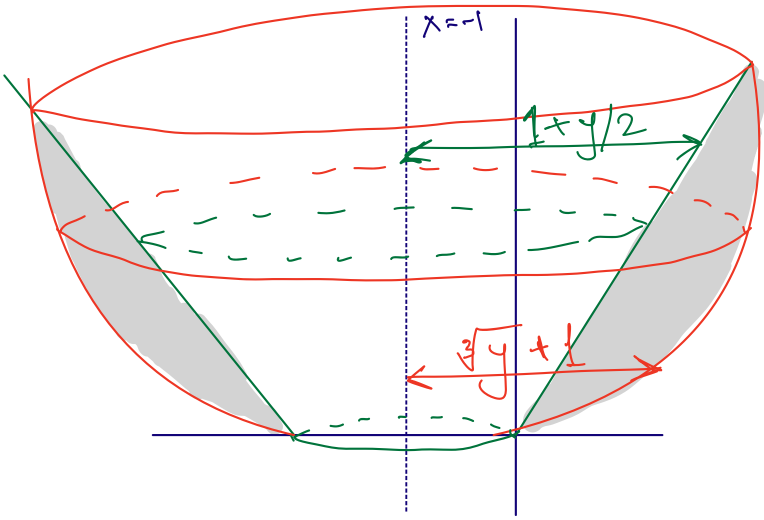
(c) Revolve S about the line $x = -1$.

Write in terms of y:

$x = y^{1/3} \quad x = y/2$

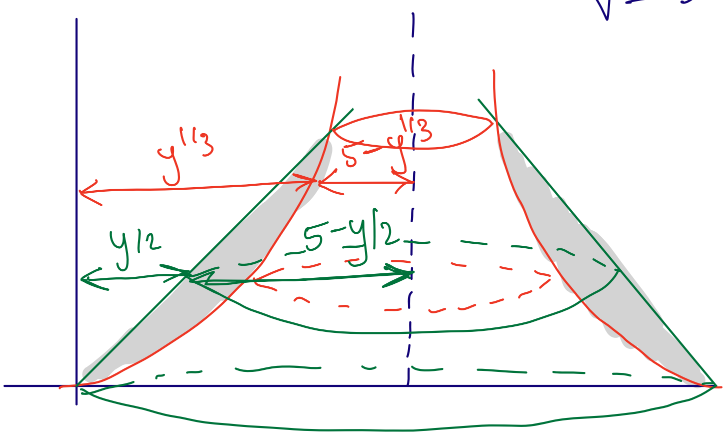
Intersection: $y = 2\sqrt{2}$

$$V = \pi \int_0^{2\sqrt{2}} \left(y^{1/3}+1\right)^2 - \left(y/2+1\right)^2 dy$$



(d) Revolve S about the line $x = 5$.

$$V = \pi \int_0^{2\sqrt{2}} \left(5-y/2\right)^2 - \left(5-y^{1/3}\right)^2 dy$$



(b) Outer radius $= 4 - x^3$
Inner radius $= 4 - 2x$

$$V = \pi \int_0^{\sqrt{2}} \left(4-x^3\right)^2 - \left(4-2x\right)^2 dx$$