§6.2 **Name:** ______________________

Fall MATH 1120 Lec003 12 September - 16 September 2022

Solids of Revolution and their Volumes

Main Concepts

- Solids of Revolution are obtained by revolving curves around a fixed axis.
- **Disk method:** Used to compute volumes of solids generated by revolving a single curve. Suppose $y = r(x)$ is a nonnegative continuous function on $[a, b]$, then the volume of the solid of revolution given by revolving the curve about the x-axis over the interval is given by:

$$V = \int_a^b \pi r(x)^2 \,\mathrm{d}x$$

- **Washer method:** Suppose $R(x)$ and $r(x)$ are nonnegative continuous functions on $[a, b]$ that satisfy $R(x) \geq r(x)$ for all x in $[a, b]$, the solid of revolution has volume

$$V = \int_a^b \pi[R(x)^2 - r(x)^2] \,\mathrm{d}x$$

- When the curve is revolved around a different axis, say $y = a$, the volume of the solid of revolution can be computed after shifting the curves by a and determining the radii with respect to the new axis of revolution.

Activities

Activity 6.2.2

In each of the following questions, draw a careful, labeled sketch of the region described, as well as the resulting solid that results from revolving the region about the stated axis. In addition, draw a representative slice and state the volume of that slice, along with a definite integral whose value is the volume of the entire solid. It is not necessary to evaluate the integrals you find.

(a) The region S bounded by the x-axis, the curve $y = \sqrt{x}$, and the line $x = 4$; revolve S about the x-axis.

(b) The region S bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 2$; revolve S about the x-axis.

(c) The finite region S bounded by the curves $y = \sqrt{x}$ and $y = x^3$; revolve S about the x-axis.

(d) The finite region S bounded by the curves $y = 2x^2 + 1$ and $y = x^2 + 4$; revolve S about the x-axis.

(e) The region S bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 2$; revolve S about the y-axis. How is this problem different from the one posed in part (b)?

Activity 6.2.3

In each of the following questions, draw a careful, labeled sketch of the region described, as well as the resulting solid that results from revolving the region about the stated axis. In addition, draw a representative slice and state the volume of that slice, along with a definite integral whose value is the volume of the entire solid. It is not necessary to evaluate the integrals you find.

(a) The region S bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 2$; revolve S about the y-axis.

(b) The region S bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $x = 4$; revolve S about the y-axis.

(c) The finite region S in the first quadrant bounded by the curves $y = 2x$ and $y = x^3$; revolve S about the x-axis.

(d) The finite region S in the first quadrant bounded by the curves $y = 2x$ and $y = x^3$; revolve S about the y-axis.

(e) The finite region S bounded by the curves $x = (y-1)^2$, and $y = x - 1$; revolve S about the y-axis.

Activity 6.2.4

In each of the following questions, draw a careful, labeled sketch of the region described, as well as the resulting solid that results from revolving the region about the stated axis. In addition, draw a representative slice and state the volume of that slice, along with a definite integral whose value is the volume of the entire solid. It is not necessary to evaluate the integrals you find. For each prompt, use the finite region S in the first quadrant bounded by the curves $y = 2x$ and $y = x^3$.

(a) Revolve S about the line $y = -2$.

(b) Revolve S about the line $y = 4$.

(c) Revolve S about the line $x = -1$.

(d) Revolve S about the line $x = 5$.