§8.1 NAME: ____________________

Fall MATH 1120 Lec003 31 October - 04 November 2022

SEQUENCES

MAIN CONCEPTS

- A **sequence** is a (possibly infinite) list of numbers $s_1, s_2, s_3, ...$ in specified order. Examples:
  (a) $1, 2, 3, 4, ...$
  (b) $2, 4, 6, 8, ...$
  (c) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...$
- s_n is referred to as the n^{th} term of the sequence.
- A sequence can be thought of as a function whose domain is the set of positive integers,

$$f(n) = s_n.$$

  For example:
  (a) $f(n) = n$ gives the sequence $1, 2, 3, 4, ...$
  (b) $f(n) = 2n$ gives the sequence $2, 4, 6, 8, ...$
  (c) $f(n) = \frac{1}{n}$ gives the sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...$
- When we say that a sequence $\{a_n\}$ **converges** to the number L, we mean that a_n can be made as close to L as we like by taking n to be large enough.
- **Formal definition:** We say that $\{a_n\}$ **converges to the limit** L and write

$$\lim_{n\to\infty} a_n = L$$

  if, for every $\epsilon > 0$, there exists $N > 0$ such that

$$|L - a_n| < \epsilon$$

  for all $n > N$.

Activities

Activity 1

Figure 1 shows triangular numbers. The n-th **triangular number** T_n is the number of dots arranged in an equilateral triangle with n dots on the side.

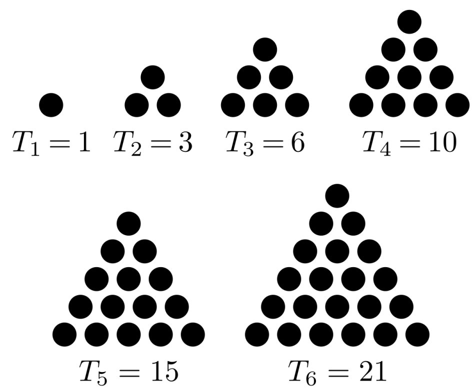

Figure 1: Triangular numbers

(a) How is T_n related to T_{n-1}? (**Hint:** try to argue using a picture with the triangles)

(b) Using the result in the previous part, find a formula involving summation for T_n. Check that your formula works for $n = 1, 2, 3$.

(c) Find a formula for T_n that does not involve a summation sign. Check that your formula works for $n = 1, 2, 3$.

Activity 2

Consider the sequence $\{T_n\}$ of triangular numbers.

(a) What is the function $f(n)$ that defines the n-th term of the sequence $\{T_n\}$?

(b) For each of the following expressions, identify if it is a number, sequence, or function:

(i) T_n

(ii) $\{T_{n+1}\}$

(iii) T_3

(iv) $\{f(n)\}$

(v) $f(n)$

(vi) $f(3)$

Activity 3

In this activity we will try to build some intuition about the definition of the limit of a sequence.

(a) Figure 2 shows a visual interpretation of the limit definition. We plot the sequence by

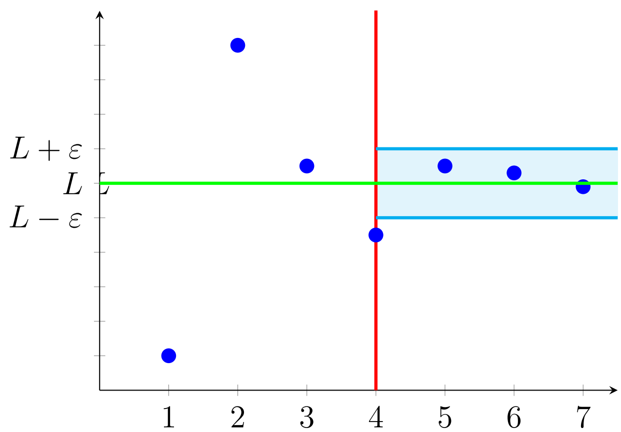

Figure 2: A visual interpretation of the limit

considering the graph $y = f(x)$ evaluated at integer values of x (so the plot consists of a bunch of disconnected points). We call the light blue region an "ε-band around L", that is, the set of points in the plane whose y-coordinate lies between $L - \varepsilon$ and $L + \varepsilon$. Then the definition of limit could be restated as follows:

$\lim_{n\to\infty} a_n = L$ *means that no matter how small* ε *is, we can always find a number* N *such that all points of the plotted sequence strictly to the right of the line* $x = N$ *lie within an* ε*-band around* L*.*

(b) For each of these parts of the original definition, find the corresponding concept on the visual one:

(i) ... for every $\varepsilon > 0$ ...

(ii) ... there exists $N > 0$ ...

(iii) ... $|L - a_n| < \varepsilon$ for all $n > N$.

(c) For this part, we'll play a game. The sequence $\{a_n\}$ is given. Player A will try to show that the limit of the sequence is L, and player B will try to disprove it.

- then player A chooses a number L
- then player B chooses a margin of error ε
- then player A chooses a number N
- then to win, player B has to find a number n larger than N such that a_n is further from L than the margin of error ε. If player B can't find such a number, player A wins.

Then the definition of a limit could be restated as follows:

If player A has a winning strategy after choosing L then the sequence converges to L.

(d) For each of these parts of the original definition, find the corresponding concept on the game one:

(i) ... for every $\varepsilon > 0$ ...

(ii) ... there exists $N > 0$ ...

(iii) ... $|L - a_n| < \varepsilon$ for all $n > N$.