§8.1 **Name:** ______________________________

Fall MATH 1120 Lec003 31 October - 04 November 2022

Sequences contd.

Review

- **Formal definition:** We say that $\{a_n\}$ **converges to the limit** L and write

$$\lim_{n\to\infty} a_n = L$$

if, for every $\epsilon > 0$, there exists $N > 0$ such that

$$|L - a_n| < \epsilon$$

for all $n > N$.

Main Concepts

- **Theorem 1** If $\lim_{x\to\infty} f(x)$ exists, then the sequence $a_n = f(n)$ converges to the same limit

$$\lim_{n\to\infty} a_n = \lim_{x\to\infty} f(x).$$

- **Theorem 2** If f is continuous and $\lim_{n\to\infty} a_n = L$, then

$$\lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right) = f(L).$$

In other words, we may pass a limit of a sequence inside a continuous function.

Activities

Activity 1

Let $a_n = \frac{1+2n}{3n-2}$.

(a) Can you guess what $\lim_{n\to\infty} a_n$ is? (**Hint:** try to plug in a very large number for n)

(b) Compute $\left|a_n - \frac{2}{3}\right|$ and simplify it as much as possible.

(c) Consider your expression for $\left|a_n - \frac{2}{3}\right|$ from the previous part. Can you make it small by increasing n?

(d) How large would n have to be to ensure that $\left|a_n - \frac{2}{3}\right| < \varepsilon$?

(e) Explain how this proves that $\lim_{n\to\infty} a_n = \frac{2}{3}$, with all three interpretations (the definition, the visual one, and the game one).

Activity 2

For each of the following sequences, find a function defining them and use Theorem 1 to find their limits.

(a) Let $\{a_n\}$ be the sequence starting $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots$.

(b) Let $b_n = \frac{n+\ln(n)}{n^2}$ (**Hint:** you might want to use L'Hôpital's rule)

Activity 3

Let $f(x) = \sin(x\pi)$ and let $c_n = f(n)$.

(a) What is $\lim_{x\to\infty} f(x)$?

(b) What is $\lim_{n\to\infty} c_n$?

(c) Does this contradict Theorem 1? Explain.

Activity 4

In this activity we will compute $\lim_{n\to\infty} a_n$, where $\{a_n\}$ is the sequence

$$1, \sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, \ldots$$

(a) Find a function $f(x)$ such that $f(n) = a_n$.

(b) Let $g(x) = \ln(f(x))$. Find

$$\lim_{x\to\infty} g(x)$$

(c) Consider the following argument

We found that $\lim_{x\to\infty} g(x) = 0$*. We also know that* $g(x) = \ln(f(x))$*, and* $\ln(x)$ *is continuous so therefore we have*

$$0 = \lim_{x\to\infty} g(x) = \lim_{x\to\infty} \ln(f(x)) = \ln\left(\lim_{x\to\infty} f(x)\right)$$

therefore, since $\ln(1) = 0$*, by Theorem 2, we can conclude that* $\lim_{x\to\infty} f(x) = 1$.

Is this argument correctly applying Theorem 2?

(d) Can you fix the argument in the previous part so that it correctly uses Theorem 2?