§8.2 **Name:** SOLUTIONS

Fall MATH 1120 Lec003 7 November - 11 November 2022

Geometric Series

Main Concepts

- A **Geometric Sequence** (a.k.a. Geometric Progression) is a sequence of the form $a_n = ar^{n-1}$, where a and r some given numbers (r is often referred to as the *ratio*). A typical geometric sequence looks like

$$a,\ ar,\ ar^2,\ ar^3,\ \ldots$$

- Examples:

$$a = 1\ r = \tfrac{1}{2}:\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \ldots$$

$$a = 3\ r = 2:\ 3,\ 6,\ 12,\ 24,\ \ldots$$

$$a = \pi\ r = -\tfrac{1}{3}:\ \pi,\ -\frac{\pi}{3},\ \frac{\pi}{9},\ -\frac{\pi}{27},\ \frac{\pi}{81},\ \ldots$$

- The n^{th}**-Geometric Partial sum** is the sum of the first n terms of a geometric sequence. It is denoted by S_n and

$$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} = \sum_{k=1}^{n} ar^{k-1}$$

- If we know a and r, and $r \neq 1$ then S_n is given by the formula:

$$S_n = \frac{a(1-r^n)}{1-r}$$

- The **Infinite Geometric Series** is the sum of all infinite terms in a given geometric sequence i.e.

$$S = a + ar + ar^2 + ar^3 + \ldots = \sum_{k=1}^{\infty} ar^{k-1}$$

- The infinite geometric series only exists (converges) when $|r| < 1$, otherwise it does not exist (diverges). When it converges (i.e. when $|r| < 1$), the value of the infinite series is given by

$$S = \frac{a}{1-r}$$

Activities

Activity 8.2.2

Let a and r be real numbers (with $r \neq 1$) and let

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1}.$$

In this activity we will find a shortcut formula for S_n that does not involve a sum of n terms.

(a) Multiply S_n by r. What does the resulting sum look like?

$$rS_n = ar + ar^2 + ar^3 + \ldots + ar^n$$

(b) Subtract rS_n from S_n and explain why

$$S_n - rS_n = a - ar^n.$$

$$S_n - rS_n = a + \cancel{ar} + \cancel{ar^2} + \ldots + \cancel{ar^{n-1}}$$
$$- (\cancel{ar} + \cancel{ar^2} + \ldots + \cancel{ar^{n-1}} + ar^n)$$

$$= a - ar^n$$

(c) Solve the equation above for S_n to find a simple formula for S_n that does not involve adding n terms.

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Activity 8.2.3

Let $r \neq 1$ and a be real numbers and let

$$S = a + ar + ar^2 + \ldots + ar^{n-1} + \ldots$$

be an infinite geometric series. For each positive integer n, let

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1}.$$

(a) What should we allow n to approach in order to have S_n approach S?

n must approach ∞

(b) What is the value of $\lim_{n\to\infty} r^n$ for $|r| > 1$? for $|r| < 1$? Explain.

$$\lim_{n\to\infty} r^n = \begin{cases} \infty & |r|>1 \\ 0 & |r|<1 \end{cases}$$

(c) If $|r| < 1$, use the formula for S_n and your observations in (a) and (b) to explain why S is finite and find a resulting formula for S.

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a(1-r^n)}{1-r}$$

$$= \frac{a}{1-r} \quad \text{if } |r|<1$$