§1.1, 1.2, 1.3 **Name:** ____________________

Spring MATH 2220 22 January 2020

1 Review

1.1 Vectors in Two and Three-Dimensional Space

- **Basic vector operations:** addition and scalar multiplication.
- **Standard bases**: The unit vectors $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, $\mathbf{k} = (0, 0, 1)$ correspond to the standard basis vectors for the x-, y-, and z-axes, respectively.
- **Parametric equation of a line** through the point $\mathbf{a}$ (based at the origin) in the direction of $\mathbf{v}$ (based at $\mathbf{a}$) is $l(t) = \mathbf{a} + t\mathbf{v}$.

1.2 The Inner Product, Length, and Distance

- The **inner product** (or **dot product**) of the vectors $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$ is defined as

$$\begin{aligned}\langle \mathbf{a}, \mathbf{b}\rangle = \mathbf{a}\cdot\mathbf{b} &= a_1b_1 + a_2b_2 + a_3b_3 \\ &= ||\mathbf{a}||\,||\mathbf{b}||\cos\theta.\end{aligned}$$

  where $||\mathbf{a}|| = \sqrt{\mathbf{a}\cdot\mathbf{a}}$ is the **length** or **norm** of $\mathbf{a}$ and θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$.

- The **orthogonal projection** of $\mathbf{v}$ onto $\mathbf{a}(\neq \mathbf{0})$ is

$$\mathbf{p} = \frac{\mathbf{a}\cdot\mathbf{v}}{||\mathbf{a}||^2}\,\mathbf{a}.$$

1.3 Matrices, Determinants, and the Cross Product

- The **cross product** of the vectors $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$ is defined as

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

  and the length of $\mathbf{a}\times\mathbf{b}$ is $||\mathbf{a}\times\mathbf{b}|| = ||\mathbf{a}||\,||\mathbf{b}||\sin\theta$, where θ is the angle with $0 \leq \theta \leq \pi$ between $\mathbf{a}$ and $\mathbf{b}$.

- The **equation of a plane** through the points (x_0, y_0, z_0) and normal to the vector $\mathbf{n} = (A, B, C)$ is $A(x - x_0) + B(y - y_0) + c(z - z_0) = 0$.

Practice Problems

1. Find the distance from $(2, 8, -1)$ to the line that passes through $(1, 1, 1)$ in the direction of the vector $(1/\sqrt{3})\mathbf{i}+(1/\sqrt{3})\mathbf{j}+(1/\sqrt{3})\mathbf{k}$.

2. Sketch and compute the volume of the parallelepiped spanned by:

$$\mathbf{u} = \langle 2, 2, 1\rangle, \quad \mathbf{v} = \langle 1, 0, 3\rangle, \quad \mathbf{w} = \langle 0, -4, 0\rangle$$

3. Which of the following is a parametrization of the line through $P = (4, 9, 8)$ perpendicular to the xz-plane (Figure 1)?

   (a) $\vec{r}(t) = \langle 4, 9, 8\rangle + t\langle 1, 0, 1\rangle$

   (b) $\vec{r}(t) = \langle 4, 9, 8\rangle + t\langle 0, 0, 1\rangle$

   (c) $\vec{r}(t) = \langle 4, 9, 8\rangle + t\langle 0, 1, 0\rangle$

   (d) $\vec{r}(t) = \langle 4, 9, 8\rangle + t\langle 1, 1, 0\rangle$

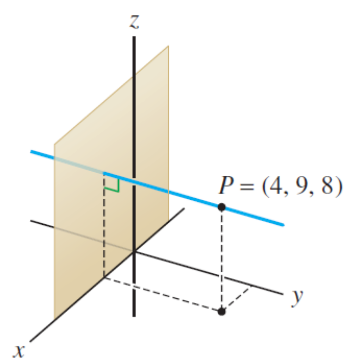

Figure 1: Question 3.

4. In this exercise we will prove the Cauchy-Schwarz inequality: If $\vec{v}$ and $\vec{w}$ are any two vectors, then

$$|\vec{v} \cdot \vec{w}| \leq \|\vec{v}\|\|\vec{w}\|$$

   (a) Let $f(x) = \|x\vec{v} + \vec{w}\|^2$ for a scalar x. Show that $f(x) = ax^2 + bx + c$, where $a = \|\vec{v}\|^2$, $b = 2\vec{v} \cdot \vec{w}$ and $c = \|\vec{w}\|^2$.

   (b) Conclude that $b^2 - 4ac \leq 0$. *Hint:* Does the quadratic polynomial have any real roots?

5. Find the equation of the plane containing the three points $P = (3, 0, 3)$, $Q = (4, 3, 2)$ and $R = (1, 1, 2)$. Find the point of its intersection with the line $\mathbf{r}(t) = \left\langle -1 + \frac{3}{2}t, 1, -t \right\rangle$.