§2.4, 2.5 **Name:** ______________________________

Spring MATH 2220 05 February 2020

1 Properties of the Derivative

1. The *Chain rule* states that

$$\mathrm{D}(f \circ g)(\mathbf{x}_0) = \mathrm{D}f(g(\mathbf{x}_0))\mathrm{D}g(\mathbf{x}_0),$$

   where $g : U \subset \mathbb{R}^n \to \mathbb{R}^m$ and $f : V \subset \mathbb{R}^m \to \mathbb{R}^p$ are differentiable functions and $g(U) \subset V$, so that composition is well defined.

2. Special cases of the chain rule:

   - Along a path $\langle x(t), y(t), z(t)\rangle$ parametrized by $t \in \mathbb{R}$, the derivative of the composition $h(t) = f(x(t), y(t), z(t))$ is given by

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\partial f}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\partial f}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}t}.$$

   - If f is a function of u, v, w which are in turn functions of x, y, z i.e. if we define the composition as

$$h(x, y, z) := f(u(x, y, z), v(x, y, z), w(x, y, z)),$$

     then the partial derivatives of h are given by

$$\frac{\partial h}{\partial t} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial x}$$

2 Practice Problems

1. Suppose $g(u)$ is a differentiable function and let $f(x, y) = g(x^2 + y^2)$. If $u = x^2 + y^2$, show that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = 4u\left(\frac{\mathrm{d}g}{\mathrm{d}u}\right)^2$$

2. A fighter plane, which can shoot a laser beam straight ahead, travels along the path $\vec{r}(t) = \langle t - t^3, 12 - t^2, 3 - t\rangle$. Show that the pilot cannot hit any target on the x-axis.

3. Use sine and cosine to parametrize the intersection of the surfaces $x^2 + y^2 = 1$ and $z = 4x^2$ (Refer Figure 1).

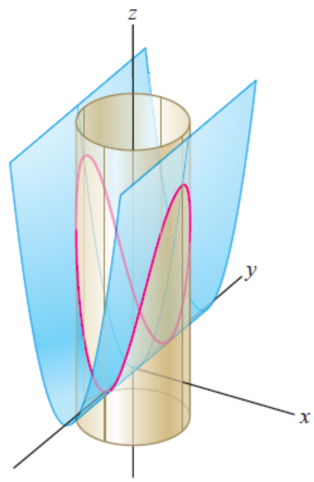

Figure 1:

4. Verify the chain rule for $f(u, v) = uv$ where

$$u(x, y) = x^2 - y^2 \quad v(x, y) = x^2 + y^2$$

5. Suppose a duck is swimming in the circle $x = \cos t, y = \sin(t)$, while the temperature $T = x^2 e^y - xy^3$. Find $\mathrm{d}T/\mathrm{d}t$:

   (a) by the Chain Rule

   (b) by expressing T in terms of t and differentiating.