§1.1,1.2,1.3 **Name:** ____________________

Fall MATH 2930 02 September 2021

Review

Differential Equations

An ordinary differential equation can be written generally as

$$F\left(t, y, \frac{\mathrm{d}y}{\mathrm{d}t}, ..., \frac{\mathrm{d}^n y}{\mathrm{d}t^n}\right) = 0$$

Classification of Differential equations

- Number of independent variables: **Ordinary** or **Partial**,

$$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}\right)^2 + \frac{\mathrm{d}y}{\mathrm{d}t} + y = \sin(t) \quad \text{vs} \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 u}{\partial x \partial y} + u^3 = 0$$

- Number of dependent variables: **Single** variable or **System** of variables,

$$t\frac{\mathrm{d}y}{\mathrm{d}t} + y = 0 \quad \text{vs} \quad \begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = x - xy \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = xy - y \end{cases}$$

- **Order**: Highest derivative occurring in the DE,

$$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}\right)^2 + \frac{\mathrm{d}y}{\mathrm{d}t} + y = \sin(t) \quad \text{vs} \quad \frac{\mathrm{d}^3 y}{\mathrm{d}t^3} + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 + y = 0$$

- **Linear** or **Nonlinear**: Depending on whether F is a linear or nonlinear function y and its derivatives,

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \frac{\mathrm{d}y}{\mathrm{d}t} + y = 0 \quad \text{vs} \quad \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \sin(y) = 0$$

Practice Problems

1. Is the function $y(t) = 1 + t$ a solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{y^2 - 1}{t^2 + 2t}$? Why or why not? What about $y = 1 + 2t$? What about $y = 1$?

2. Suppose you and a friend are interested in how the number of fish in Lake Cayuga changes over time. Luckily, you're both in this class, and you decide to use a differential equation as a model. You remember that one of the tenets of math modelling is to start with a simple model and then build up complexity as needed, so you agree to make the following simplifying assumptions:

   (a) there is only one species of fish in the lake;

   (b) the species reproduces continuously;

   (c) the species has access to unlimited resources (eg: food, space);

   (d) the species is not being preyed upon;

   (e) no fish are being added to the lake artificially (eg: fishy immigration).

   You friend has come up with a few candidate models for $P(t)$, denoting the population size at time t. For each equation, discuss whether or not it is a good model in this context, specifically in light of the above assumptions as well as normal physical constraints (eg: there can't be a negative number of fish).

(a) $\frac{dP}{dt} = (t+1)^2$

(b) $\frac{dP}{dt} = 2P$

(c) $\frac{dP}{dt} = 1 - P^2$

(d) $\frac{dP}{dt} = P^2 + 1$

3. Modelling Predator-Prey relationships:

   (a) Draw a graph with the population of fish, $P(t)$ on the y axis and time on the x axis.

(b) Suppose at $t = t_0$, that is after allowing some time to elapse, we introduce a species of shark into the lake. Let the number of sharks at time t be $Q(t)$. The sharks begin to prey on the fish already in the lake. Draw a graph showing how $P(t)$ would behave for a short period of time after t_0.

(c) As the sharks continue to prey on the fish, they reproduce in larger numbers. Draw a graph showing how $Q(t)$ would behave for a short period of time after t_0.

(d) Can the shark population grow without limits? What happens when there are too many sharks and too few fish to prey on? Draw a graph representing both $P(t)$ and $Q(t)$ over a large period of time.

(e) How would a graph with P on the x axis and Q on the y axis look?

4. Consider the differential equation

$$\frac{dy}{dt} = \frac{1}{2}\, y(2+y)(y-8).$$

For what values of y is a solution to this differential equation

(a) increasing,

(b) decreasing,

(c) constant?

In each case, explain your reasoning.

Some of the questions in this worksheet were prepared by Elliot Cartee.