§3.1,3.2,3.3,3.4 NAME: ____________

Fall MATH 2930 07 October 2021

REVIEW

2ND ORDER ODES

- A *Linear, Second order, Constant Coefficient, Homogeneous* ODE is written as

$$ay'' + by' + cy = 0 \quad y(t_0) = y_0 \quad y'(t_0) = y_0'$$

- Steps to solve the above:
  1. Plug in $y = e^{rt}$ to get the *Characteristic equation*:
  $$ar^2 + br + c = 0.$$
  2. Solve the characteristic equation (either factoring or using the quadratic formula) to get two roots: r_1 and r_2 (these can potentially be complex numbers)
  3. If $r_1 \neq r_2$, the *general solution* is given by:
  $$y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}.$$
  4. If $r_1 = r_2$ (repeated roots), the *general solution* is given by
  $$y(t) = C_1 e^{r_1 t} + C_2 t e^{r_2 t}.$$
  5. Plug in the two initial conditions to find C_1 and C_2.

COMPLEX NUMBERS

- A complex number is of the form $z = a + ib$ where $i = \sqrt{-1}$.
- *Euler's formula* is given by
$$e^{i\theta} = \cos\theta + i\sin\theta.$$
- The exponential of a complex number maybe written as
$$e^z = e^{a+ib} = e^a e^{ib} = e^a(\cos b + i\sin b)$$

REDUCTION OF ORDER

Given a differential equation,

$$y'' + p(t)y' + q(t)y = 0$$

and a solution $y_1(t)$, then a second solution, $y_2(t) = v(t)y_1(t)$ can be found by solving

$$y_1 v'' + (2y_1' + py_1)v' = 0$$

for v.

Practice Problems

1. Example Problem: Solve the differential equation:

$$5y'' + 2y' + y = 0 \quad y(0) = 2 \quad y'(0) = 1$$

2. Suppose $y_1(t) = t^{-1}$ is a solution to the equation

$$2t^2y'' + 3ty' - y = 0 \quad t > 0.$$

Let $y_2(t) = v(t)y_1(t) = \frac{v(t)}{t}$.

   (a) Compute y_2' and y_2'' (your answer will involve v, v' and v'').

(b) Plug y_2, y_2' and y_2'' into the differential equation and simplify to get an ODE for $v(t)$.

(c) Solve the resulting ODE for $v(t)$ and hence find $y_2(t)$.

3. On the way to Collegetown, a MATH 2930 student found a piece of paper on the ground. Much of the ink was smudged by water but clearly visible was the word "solution" followed by the expression,

$$y(t) = e^{-2t}(C_1 \sin(t) + C_2 \cos(t)).$$

Also visible (very conveniently) was the phrase "2nd order constant coefficient" but the differential equation itself was smudged except for

$$... + 5y = 0$$

Find the differential equation.

4. Solve the initial value problem

$$9y'' - 12y' + 4y = 0 \quad y(0) = 2 \quad y'(0) = -1.$$

Problems in this worksheet are adapted from `https://math.uchicago.edu/~ecartee`