§3.5,3.7 **Name:** ____________________

Fall MATH 2930 14 October 2021

Review

2^{nd} Order Non-homogeneous ODEs

- A linear second order non-homogeneous ODE with constant coefficients can be written as
$$ay'' + by' + cy = g(t) \qquad (*)$$
- The corresponding homogeneous ODE is
$$ay'' + by' + cy = 0 \qquad (**)$$
- If $y_1(t)$ and $y_2(t)$ are the solutions of $(**)$ (a.k.a. *homogeneous or complementary solutions*), and $Y(t)$ is any solution of $(*)$(a.k.a. *particular solution*), then the *general solution* of $(*)$ is written as
$$y(t) = c_1y_1(t) + c_2y_2(t) + Y(t)$$
- If Y_1 is a particular solution of $ay'' + by' + cy = g_1(t)$ and Y_2 is a particular solution of $ay'' + by' + cy = g_2(t)$, then $Y_1 + Y_2$ is a solution of $y'' + P(t)y' + q(t)y = g_1(t) + g_2(t)$.

Undetermined Coefficients

- This is a method to systematically guess a particular solution for $(*)$.
- Only works when $g(t)$ is a sin, cos, exponential, polynomial or a combination of these functions.
- Steps:
  - Solve the homogeneous equation to obtain y_1 and y_2.
  - Make a guess for $Y(t)$. Common initial guess:

$g(t)$:	Guess $Y(t)$ as
t	$At + B$
t^2	$At^2 + Bt + C$
t^3	$At^3 + Bt^2 + Ct + D$
e^{at}	Ae^{at}
$\sin(at)$	$A\sin(at) + B\cos(at)$
$\cos(at)$	$A\sin(at) + B\cos(at)$
Product of functions listed above	Product of guesses listed above

- If your guess matches 1 of the homogeneous solutions, then multiply your guess by t.
- If your guess matches 2 of the homogeneous solutions, then multiply your guess by t^2 (this happens when your homogeneous equation has repeated roots).
- Plug in your guess into $(*)$ and solve for all the constants A, B, C, ... etc. to obtain $Y(t)$.
- If the $g(t)$ is the sum of several terms, do the above steps for each of those terms and add the solutions to get the full particular solution.
- Write down the general solution (by adding the homogeneous solution and particular solution).

Practice Problems

1. Example Problem: Find the general solution of

$$y'' - y' + \frac{1}{4}y = e^{t/2}$$

   Using the method of undetermined coefficients.

2. For the following non-homogeneous equations, determine a suitable form for the particular solution $Y(t)$ using the method of undetermined coefficients. **You don't have to find the coefficients!**

   (a)
$$y'' + 2y' + 5y = t$$

(b)

$$y'' + 2y' + 5y = 2te^{-2t}\cos(t)$$

(c)

$$y'' + 2y' + 5y = 3te^{-t}\cos(2t)$$

3. Solve the initial value problem

$$y'' + y' - 2y = 2t \quad y(0) = 0 \quad y'(0) = 1$$

using the method of undetermined coefficients.

Problems in this worksheet are adapted from `https://math.uchicago.edu/~ecartee`