§4.1,4.2,4.3 **NAME:** ______________________

Fall MATH 2930 28 October 2021

REVIEW

HIGHER ORDER ODES

- Very generally, an n^{th} order linear, constant coefficient ODE is written as

$$L[y] = a_0 y^{(n)} + a_1 y^{(n-1)} + ... + a_{n-1} y' + a_n y = g(t).$$

  Here, $a_0, ..., a_n$ are numbers and $a_n \neq 0$.

- If $g(t) = 0$, we call the ODE *homogeneous*. Otherwise it is inhomogeneous/non-homogeneous.

- Just as before, to solve a homogeneous ODE, we first substitute e^{rt} to get the characteristic polynomial equation,

$$a_0 r^n + a_1 r^{n-1} + ... + a_{n-1} r + a_n = 0.$$

- The above equation has n roots (some may repeat). In general, it is difficult to solve this equation (factor the polynomial) exactly. If it is possible, and the roots are $r_1, ..., r_n$, we have:

  1. For each **non-repeated** root, r, then the solution corresponding to that root is ce^{rt}, where c is an arbitrary constant.
  2. Suppose the root r **repeats** s number of times, then the corresponding solution is $c_1 e^{rt} + c_2 t e^{rt} + ... + c_s t^s e^{rt}$.
  3. For **complex** roots (they always appear in conjugate pairs), $r_\pm = a \pm ib$ the corresponding root is $e^{at}(c_1 \cos bt + c_2 \sin bt)$.

- For inhomogeneous equations, the method of undetermined coefficients can be used to find a particular solution (just like in the 2^{nd} order case).

Practice Problems

1. Example problem: Find the general solution to the following differential equation:

$$y^{(4)} - 2y^{(3)} + 2y'' - 2y' + y = e^t + \cos 2t$$

2. We are given that the homogeneous solution to a linear constant coefficient ODE is

$$y = c_1e^t + c_2te^t + c_3t^2e^t + c_4e^{-t} + e^t(c_5 \cos t + c_6 \sin t).$$

Find the differential equation.

3. Find the general solution to the following ODEs:

(a) $y^{(4)} + 2y'' + y = 0$

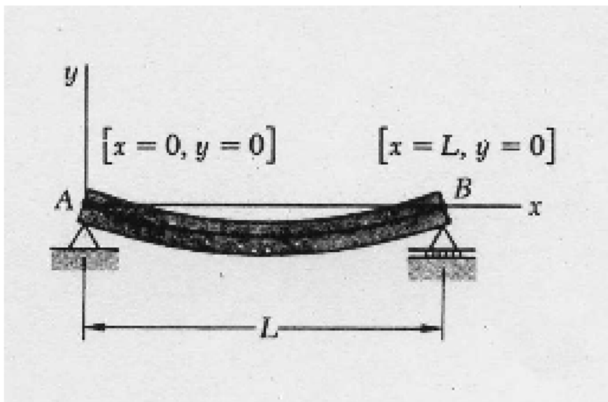
(b) $y^{(6)} - y'' = 0.$

4. Consider a horizontal metal beam of length L subject to a vertical load $f(x)$ per unit length. The resulting vertical displacement in the beam $y(x)$ satisfies a differential equation of the form

$$A\frac{d^4y}{dx^4} = f(x)$$

where A is a constant. Suppose that $f(x)$ is just a constant k, i.e., the equation is simply

$$A\frac{d^4y}{dx^4} = k.$$



(a) Find the general solution of this non-homogeneous fourth order equation.

(b) Solve the above equation for the boundary condition

$$y(0) = y''(0) = y(L) = y''(L) = 0.$$

(c) Solve the above equation for the boundary condition

$$y(0) = y'(0) = y''(L) = y'''(L) = 0.$$

Some problems in this worksheet are adapted from `https://math.uchicago.edu/~ecartee`