Prelim Review **Name:** ______________________

Fall MATH 2930 04 November 2021

1. Consider the following differential equation:

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = g(t)$$

(a) Find the homogeneous/complementary solution (i.e. $g(t) = 0$)

(b) Suppose $g(t) = e^t \sin(2t)$. Find the particular solution. What is the steady state solution?

(c) What would the guess for the particular solution be if $g(t) = e^{3t}\sin(2t)$ instead? (**No need to find the undetermined coefficients**).

2. Consider the two coupled differential equations:

$$u_1'' + 5u_1 = 2u_2 \qquad u_2'' + 2u_2 = 2u_1$$

Here, 'coupled' means that the differential equation for u_1 has u_2 terms and vice versa.

(a) Express the two 2nd order equations above as a single 4th order equation involving only u_1. *Hint: Try to express u_2 in terms of u_1 using the second equation and plug it into the first.*

(b) Find the general solution to the 4th order equation you found above with the initial conditions

$$u_1(0) = 1 \quad u_1'(0) = 0 \quad u_2(0) = 2 \quad u_2'(0) = 0.$$

3. Assume that the system described by the equation

$$mu'' + \gamma u' + ku = 0$$

is critically damped and that the initial conditions are $u(0) = u_0$, $u'(0) = v_0$.

(a) If $v_0 = 0$ and $u_0 \neq 0$ show that $u \to 0$ as $t \to \infty$ but that u is never zero.

(b) Assuming $u_0 > 0$, determine a condition on v_0 that will ensure that the mass passes through its equilibrium once released.

4. A car and its suspension system are idealized as a damped spring-mass system, with natural frequency 0.5Hz and damping coefficient 0.2. Suppose the car drives at speed V over a road with sinusoidal bumps. Suppose the distance between two bumps is 10m and the height of a bump 20cm. At what speed does the maximum amplitude vibration occur and what is the corresponding vibration amplitude? (see figure)